# 2D Boolean Control Networks: A Progress Report

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## Abstract

1D Boolean Control Networks (BCN's), and in particular 1D Boolean Networks (BN's), have recently witnessed a large interest as effective tools for investigating a number of biological phaenomena and technological models, whose variables display a finite number of operation levels.

The purpose of this paper is to discuss some problems that arise when an extension of 1D BCN theory to a 2D environment is endeavored. 2D BCN's are 2D systems whose (local) states $x(i,j)$, inputs $u(i,j)$ and outputs $y(i,j), i,j \in \mathbb{Z}$ take values in three finite alphabets $X, U$ and $Y$, and update according to the following equations

$$x(i+1,j+1) = f(x(i,j+1), x(i+1,j), u(i,j+1), u(i+1,j)) \quad (1)$$
$$y(i,j) = h(x(i,j) \quad (2)$$

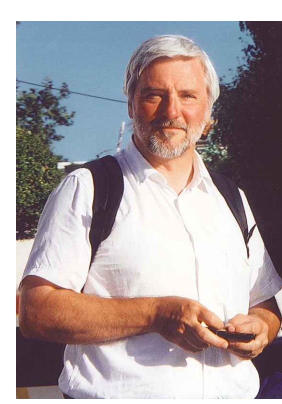
Here $f : X \times X \times U \times U \to X$ and $h : X \to Y$ are arbitrary maps.

In particular, when the state dynamics (1) is autonomous, we are faced with a 2D BN. The global states $\chi_\nu = \{x(i, i+\nu), i \in \mathbb{Z}\}$ can be viewed as shift spaces over the local state alphabet $X$ and the evolution map

$$\phi : \chi_\nu \to \chi_{\nu+1} \quad (3)$$

is the sliding block code induced by the 2-block map $x(i+1,j+1) = f(x(i,j+1), x(i+1,j))$. Asymptotic stability, i.e. the convergence of (3) to a unique constant sequence of local states, say $\bar{\chi} = (...0,0,0,...\}$, in a finite number of steps and independently of the initial global state $\chi_0$, is discussed and related to the behavior of a series of infinitely many identical 1D BCN's.

## About the Speaker:



Ettore Fornasini received the "Laurea" degree in Electronic Engineering and in Mathematics in 1969 and in 1973, respectively, from the University of Padova, Italy. In 1972 he joined the Department of Information Engineering (formerly Istituto di Elettronica e di Elettrotecnica) of the University of Padova, where he taught Electrical Network Theory, Automatic Control, System Theory, Multivariable Systems (polynomial methods) and Positive Systems Theory. He has also been responsible for lectures and courses of doctoral schools in Italy, Portugal and Austria. In 1980 E.Fornasini was appointed full professor of Mathematical Systems Theory. From 1993 to 2001, he chaired the Board of the Degree in Computer Engineering and in Electronic Engineering, and coordinated the committee for the reform of studies in the courses of Information Engineering. In 2002-05 and 2005-08 he has been elected dean of the Faculty of Engineering and in 2009-12 member of the Administration Board of the University of Padova.

He spent research periods at the Center for Mathematical System Theory, Gainesville (Florida), at the MIT Laboratory for Information and Decision Systems, Cambridge (Massachusetts), at the Technical University of Delft (The Netherlands), at the University of Innsbruck (Austria), at the University of Aveiro (in 2010, 13, 14) and has been invited to give lectures at various foreign universities: Wurzburg, Prague, Aveiro, Delft, Innsbruck, Pennsylvania State University, etc. He has been associate editor of "Circuit Systems and Signal Processing" and editor of "Multidimensional Systems and Signal Processing". He also exercised functions as a referee of research projects of the NSF and the research projects funded by MURST.

E.Fornasini is the author of over one hundred and sixty scientific papers in the field of Dynamical Systems Theory and Control, which were published in international journals or presented, often by invitation, at international conferences. His first research interests included the realization problem of bilinear systems, the structure of dissipative dynamical systems, and the connectivity properties of linear systems. In 1973 he began to collaborate with G.Marchesini on the theory of multidimensional systems: his joint work with G.Marchesini and, subsequently, with M.Bisiacco, M.Sebek, S.Zampieri, P.Rocha, M.E.Valcher, R.Pinto, and T.Pinho allowed to obtain several results concerning realization, stability, feedback stabilizability, failure detection, noninteracting and optimal control of 2D systems, 1D and nD coding theory, 2D Markov chains and their asymptotic behavior, 2D positive systems, with particular reference to the algebraic aspects of the pairs of non-negative matrices, nD behavior theory, and some applications of 2D systems to river pollution and traffic modelling. In the last few years, E.Fornasini collaborated with M.E.Valcher to the investigation of positive switched dynamical systems and boolean control networks.